

# Some Remarks on the Measure of Effectiveness of Freight Transport

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This paper includes a definition of the indicator of the effectiveness of spatial trade on the macro scale. The degree of efficiency of a given sector, for example transportation, is determined by the relation of production and its cost. This relationship can be evaluated by various models of spatial economies. One of the possible way is considered in this study.

**Keywords:** freight transport, effectiveness / efficiency.

## 1. INTRODUCTION

Transport activity accompanies most economic processes. It is therefore reasonable to make economic macromodels describing spatial distributions of production and consumption of goods and services when defining performance measures of en masse transport and in the long horizon, the starting point and basis for their construction. Such models naturally have "built in" demand and supply relationships with the characteristics of the transport sector (above all the volume of transport and its costs). This paper is intended as a preliminary theoretical study, which aims to propose a measure of the efficiency of the transport sector. For this purpose, we will use a model of the economic system located in a number of spatially separated places, called regions, and with the transport sector responsible for the transfer of goods and services between those regions. The model discussed in the following is similar to the class models initiated by P. Krugman (Helpman & Krugman (1985)). The ideas used in the model below can be found in many articles by Krugman and his colleagues; from the overviews Capello (2007), Combes et al. (2008), Fujita (2005) and Neary (2004)) can be mentioned. These structures offer a way to explain the observed disparities between the manufacturing potential of economies of different countries / regions, highlighting the role of such factors as the

availability and cost of high and lower qualified work, its mobility, the availability of intermediate goods as well as the cost of goods exchange, hence the costs of transporting goods. The cost of transport and its changing share in the price of manufactured goods can make the process, in some conditions, a consolidating factor; in other situations it acts in favour of centrifugal tendencies. In a great simplification, this means that high transport costs favour consolidation of production and its location where there is sufficient workforce and adequate supply of other inputs, and a sufficiently large market for finished goods. The lower cost of transport is an argument for locating production in the region with resources of production factors (e.g. high supply of cheaper labour). Observation of these trends and their changes over time allows to capture changes in the relation of transport costs and prices of manufactured goods, and consequently to draw conclusions about the efficiency of the transport sector in the economy. The model under discussion is only short-term: we do not take into account changes in the spatial distribution of productive forces. The changing share of prices of transport services in the prices of manufactured goods as well as consumer spending fluctuations may thus provide a starting point for analyzing the efficiency of the transport sector.

2. FORMULATION OF THE MODEL AND A DEFINITION OF THE SUBJECT MEASURE

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Let us consider an economic system consisting of 2 regions (states) with a network of connections for the physical exchange of goods 1. We will distinguish, among the market players in each region, (infinitely numerous) groups: producers of intermediate goods, producers of finished goods and consumers. We further characterize the listed. We further characterize the listed. Each of the intermediate goods is produced under monopoly conditions by another manufacturer located in one or the other region. Individual types of intermediate goods will be indexed by an indicator  $i$ , while the standard assumption, that we assume is that a continuum of such goods is produced (then, without decreasing generality, they can be numbered with an index  $i \in I = [0,1]$ ). Part of the region's production combined with imported goods is used locally in the production or consumption process, while the rest is exported. The production profile of intermediate goods will be described by the function

$$x : I \rightarrow R^4, x(i) = (x_{11}(i), x_{12}(i), x_{21}(i), x_{22}(i)),$$

where further components  $x(i)$  correspond to quantities 2 goods  $i$  produced locally in region 1, exported to region 2, imported from region 2 and produced in region 2. Aggregators are another category of market players. Their activity is reduced to the purchase and processing of intermediate goods in the finished good. This good can be considered as a kind of aggregate of intermediate goods being the subject of consumption. From a formal point of view, the process of manufacturing ("fitting") of such good is most often described by the operator, that assigns respectively the amount of the finished goods to the production profile of intermediate goods. Like most of the works, we use the Dixit-Stiglitz aggregator. In order to increase transparency of the record, we will focus on region 1; considerations for the next one can be formulated analogously. The total value of intermediate goods produced and consumed in the region in question is equal  $\int p_1 x_{11}$ , where  $p_1 = p_1(i)$ ,  $i \in I$ , is the price profile of intermediate goods in region 1. On the other hand,

the cost of imported goods purchased in sector 2 is equal to  $\int \hat{p}_2 x_{21}$ , where  $\hat{p}_2 = \hat{p}_2(i)$  and  $i \in I$ , is the profile of prices of the goods imported from region 2. We have  $\hat{p}_2 > p_2$ , that is, the price paid by the importer is higher than the price paid on the local market, i.e. in region 2. The direct cause of this is transport costs. In the process of import / export, the obvious role is for carriers: they are responsible for transport, their transport rates affect the price relationships in both regions. The impact of transport costs on spatial price distribution can be modelled in many ways, although the most common approach is additive or multiplicative. We will be limited to the latter. In this case, the technological factor ( $\tau = \tau(i)$ ,  $i \in I$ ) plays a fundamental role. The value of  $\tau(i)$  defines the amount of cargo - goods  $i$  - which must be dispatched from region 2 to make the unit of destination 4 in the destination region. This value emphasizes, as a matter of fact, the technological aspects

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1 The fact of considering only two regions is dictated solely by the desire to increase transparency of the record.

2 To simplify the record we will assume that the quantities of different simple goods are expressed in the same units.

2

of transport, so it depends not only on the means (or means) of transport and the route chosen, but on all aspects of logistic nature. In this case  $p_2 / p_2 = \tau$ . The cost of transport depends obviously on the type of cargo, so  $\tau$  does not have to be a constant function. Consumers staying in each region form a homogeneous, infinitely numerous, population of preferences described by the utility function depending on the current quantity of compound commodity produced. So if the formula (Dixit & Stiglitz (1977)) assumes the constant flexibility of substitution between locally produced and imported goods, then the utility of the consumer in region 1 depends on the aggregate of form:

$$m = \left( \int (x_{11}^\rho + x_{21}^\rho) \right)^{1/\rho}. \tag{1}$$

Thus indirect goods are not generally excellent substitutes; the flexibility of their substitution is  $1/(1-\rho)$ , where  $0 < \rho < 1$  does not depend on

the kind of good. Demand size is the solution to maximize utility<sup>5</sup>  $U = U(m)$

$$\max_{x_{11}, x_{21}} U(m), \quad (2)$$

with budget constraint :

$$\int p_1 x_{11} + \int p_2 x_{21} = e_1, \quad x_{11} \geq 0, x_{21} \geq 0. \quad (3)$$

where  $e_1 > 0$  are the total spending on domestic and imported goods. We assume that the non-negative (almost everywhere with respect to Lebesgue's measure on I) functions  $x_{11}, x_{21}$  are integers in the power  $\rho$  and such that the value of the objective function (2) is finite, while the price profiles  $p_1, p_2 : I \rightarrow R$  and transport cost profile  $\tau : I \rightarrow R$  are defined functions significantly limited to I (i.e.  $\text{esssupmax}(p_1, p_2, \tau) < \infty$ ). The task for the second region can be formulated analogously. Assuming that the utility function is a smooth, increasing function of the first of its arguments, the conditions that must be met by the solution of the task (2, 3) are:

3 Unless otherwise stated, integration covers the scope of all intermediate goods, so it is, according to the contract, an interval unit,  $I = [0,1]$  with the Lebesgue measure there. We will skip this information to improve transparency.

4 The value  $\tau$  ( $\tau$  1) can be understood as the material loss generated by the transport process. This is related to the concept of Dixit & Stiglitz (1977) of the way of taking into account the costs of transport: the transport process consumes  $\tau$  -1 units of goods, which is - expressed in physical units - the cost of transport. Emphasized by the authors, similarity with the process of transporting the melting mountain of ice seems quite natural.

5 For the sake of simplicity, we overlook any other, x-independent factors affecting the level of consumer utility.

3

$$\begin{aligned} \rho U'(m) x_{11}^{\rho-1}(i) + \lambda p_1(i) + \mu_{11}(i) &= 0, \\ \rho U'(m) x_{21}^{\rho-1}(i) + \lambda \tau(i) p_1(i) + \mu_{21}(i) &= 0, \\ \lambda &\in R, \\ \mu_{11}(i) x_{11}(i) &= 0, \quad \mu_{11}(i) \geq 0, \\ \mu_{21}(i) x_{21}(i) &= 0, \quad \mu_{21}(i) \geq 0, \end{aligned}$$

while the last two terms apply to intermediate goods, which means that the formulas must be fulfilled for almost all (in Lebesgue measure sense)  $i \in I$ . Assuming that the spatial distribution of generating capacities is sufficient to satisfy potentially reported demand, solving the task (2, 3) - if exists - defines the demand functions ( $s, s' = 1, 2$  denote the region numbers):

$$x_{ss'}^* = x_{ss'}^*(p_1, p_2, \tau).$$

These figures, as well as the other characteristics of the economy designated with their participation, will be used to calculate the measure of the efficiency of the transport sector. We will now go on to define the title measure of effectiveness of the transport sector in the model under consideration. The solution of task (2, 3) allow, among other things, to define the following characteristics of domestic ( $s = s'$ ) and international ( $s \neq s'$ ) flows:

$$M_{ss'}(p_1, p_2, \tau) = \int x_{ss'}^*(p_1, p_2, \tau), \quad (4)$$

$$W_{ss'}(p_1, p_2, \tau) = \int p_s x_{ss'}^*(p_1, p_2, \tau), \quad (5)$$

$$\hat{W}_{ss'}(p_1, p_2, \tau) = \int p_s \tau_{ss'} x_{ss'}^*(p_1, p_2, \tau). \quad (6)$$

The first category is the volume of goods flows, the other two are the values of flows measured in producer and importer prices. Of course, this list is not full and, if needed, may be supplemented by additional values. These variables depend in particular on the transport costs described by the function  $\tau$ . We have already mentioned that different intermediate goods have different transportability and that, for this reason, their contribution to transport costs in the manufacturer's price may be, and it is, different. The function  $\tau$  is then not, in general, a constant case. However, its "average" value may be a measure of the efficiency (primarily cost-effectiveness) of transport functioning. This value - marked with a symbol  $\tau^*$  would be that value of constant function  $\tau(i) \equiv \tau^* = \text{const} \geq 1$ , to which the characteristics  $M_{ss'}(p_1, p_2, \tau^*), W_{ss'}(p_1, p_2, \tau^*), \hat{W}_{ss'}(p_1, p_2, \tau^*)$  were most close to the values (4-6). In other words, the value of the indicator would be the solution of the task.

$$\tau^* = \tau^*(p_1, p_2, \tau) = \operatorname{argmin}\{d(\xi(p_1, p_2, \tau), \xi(p_1, p_2, y)) : y \geq 1\}, \quad (7)$$

or

$$\tau^* = \tau^*(p_1, p_2, \tau, \xi^{emp}) = \operatorname{argmin}\{d(\xi^{emp}, \xi(p_1, p_2, y)) : y \geq 1\}, \quad (8)$$

where  $p_1, p_2$  price profiles are established,  $\xi$  is a vector of components, computed using formulas (4-6),  $\xi^{emp}$  is a vector of empirical values (4-6) and  $d$  is a measure of conformity, i.e. determined by an arbitrarily chosen metric of Euclidean space:  $d(a, b) = \|a - b\|$ .

4

In each case we require that  $\tau^* \geq 1$ ; the smaller the value, the more the transport sector can be regarded as effective from the point of view of the measure under consideration. It is worth emphasizing that the greatest informative value is not so much the value of the indicator for a single period, as its changes over a longer time horizon. In the first case, the results may be significantly affected by model specification errors (e.g. form of utility function). Estimation of  $\tau^*$  over a longer period and at constant prices helps to increase the degree of reliability of the results.

### 3. THE EXAMPLE

As already mentioned the defined measure can be seen as averaged (after all intermediate goods) transport effectiveness. This thesis can be illustrated by considering the special case where consumer utility is described by the Cobb-Douglas function. If we assume that in (2) the objective function has the form

$$U(m) = m^\rho = \int (x_{11}^\rho + x_{21}^\rho).$$

the solution of the task (2, 3) satisfies in this case almost all  $i \in I$  equations:

$$\begin{aligned} m^{1/\rho-1} x_{11}(i)^{\rho-1} + \lambda p_1(i) &= 0, \\ m^{1/\rho-1} x_{21}(i)^{\rho-1} + \lambda p_2(i) \tau(i) &= 0. \end{aligned}$$

The solution is:

$$\begin{aligned} x_{11}^*(i) &= e_1 p_1(i)^{1/(\rho-1)} P_1^{\rho/(1-\rho)}, \\ x_{21}^*(i) &= e_1 p_2(i)^{1/(\rho-1)} \tau(i)^{1/(\rho-1)} P_1^{\rho/(1-\rho)}, \end{aligned}$$

where  $P_1$  is the price index (price of compound commodity) in region 1:

$$P_1 = \left( \int (p_1^{\rho/(1-\rho)} + (p_2 \tau)^{\rho/(1-\rho)}) \right)^{(\rho-1)/\rho}.$$

Values (4-6) are equal in the situation under consideration

$$\begin{aligned} M_{21}(p_1, p_2, \tau) &= e_1 P_1^{\rho/(1-\rho)} \int p_2^{1/(\rho-1)} \tau^{1/(\rho-1)}, \\ W_{21}(p_1, p_2, \tau) &= e_1 P_1^{\rho/(1-\rho)} \int p_2^{\rho/(1-\rho)} \tau^{1/(\rho-1)}, \\ \hat{W}_{21}(p_1, p_2, \tau) &= e_1 P_1^{\rho/(1-\rho)} \int p_2^{1/(\rho-1)} \tau^{\rho/(1-\rho)}. \end{aligned}$$

Let us assume the following price profiles  $p_1, p_2$  of intermediate goods in both regions: 6

$$\begin{aligned} p_1(i) &= {}_{[0,1/4]}(i) + 2 \cdot {}_{[1/2,3/4]}(i), \\ p_2(i) &= {}_{(1/4,1/2)}(i) + 3 \cdot {}_{(3/4,1)}(i). \end{aligned}$$

6 Symbol  $1_A$  denotes the characteristic function (index) of set  $A \subset I$ , that is:  $1_A(i) = \begin{cases} 1 & \text{if } i \in A, \\ 0 & \end{cases}$ .

otherwise, if  $p_1(i) = 0$  for some  $i$ , then it corresponds to a situation in which the good  $i$  is not produced in region 1. A similar convention applies in the case of  $p_2$  prices profile.

5

In the calculation of the value of the measure  $\tau^*$  we will adopt formula 7, while the match compliance criterion will be minimization of Pythagorean distance between

$$\begin{aligned} \xi(p_1, p_2, \tau) &= (M_{21}(p_1, p_2, \tau), W_{21}(p_1, p_2, \tau), \\ &\hat{W}_{21}(p_1, p_2, \tau)), \end{aligned}$$

$$\begin{aligned} \xi(p_1, p_2, \tau^*) &= (M_{21}(p_1, p_2, \tau^*), W_{21}(p_1, p_2, \tau^*), \\ &\hat{W}_{21}(p_1, p_2, \tau^*)), \end{aligned}$$

assuming that  $\tau^* : I \rightarrow R$  is a constant function with values from the interval  $[1, \infty)$ .

Let us assume that in the first period – called here the base period - the cost of transport to the first region, depending on the kind of intermediate goods, is 20% or 15% of the producer price:

$$\tau(i) = 1.2 \cdot 1_{[0,1/2]}(i) + 1.15 \cdot 1_{(1/2,1]}(i).$$

Calculated on the basis of formula 7, the measurement value  $\tau^*$  is shown in Figure 1 (black). As shown by the graph,  $\tau^*$  increases with increasing  $\rho$ , decreasing equivalently with increasing substitution flexibility ( $\sigma = 1/(1-\rho)$ ) of domestic and imported production. It is not surprising: the higher the mentioned flexibility, the easier it is for consumers to replace one of the goods with the other. This ease is conducive to limiting imports, and thus transport.

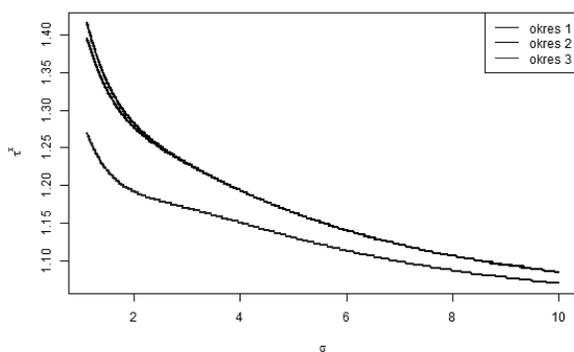


Fig. 1. Estimates of the measure  $\tau^*$  in the analyzed periods . The value of flexibility of substitution of local and imported goods, i.e.  $\sigma = 1/(1-\rho)$ , where  $0 < \rho < 1$ , was placed on the horizontal axis.

Significantly greater than the values of the meter  $\tau^*$  estimated for a single period are their fluctuations over time. They and their relationship with producers' prices are much more informative as measures of the efficiency of the transport sector. It is easy to observe when considering a hypothetical situation where in the next period, with unchanged producer prices and consumer goods expenses, transport costs have increased and in the groups of intermediate goods considered previously are respectively 35% and 3% of the producer price. In the third period these shares are equal to 30% and 20%, respectively. This corresponds to the following cost profiles  $\tau = \tau(i)$ :

$$\tau(i) = 1.35 \cdot 1_{[0,1/2]}(i) + 1.03 \cdot 1_{(1/2,1)}(i),$$

$$\tau(i) = 1.3 \cdot 1_{[0,1/2]}(i) + 1.2 \cdot 1_{(1/2,1)}(i).$$

The graph  $\tau^*$ , estimated for the second period, was highlighted in blue . In the whole range  $\rho \in [0,1)$  (i.e.  $\sigma \in (0,\infty)$ ) does not dominate over the measure calculated for the previous period: in some ranges it is larger, in the second smaller, the differences come to about 2% of the price. It is not surprising, however, that in the third period the effectiveness of transport is clearly decreasing. Producer prices remain unchanged, while the share of transport costs is rising, the average share of  $\tau^*$  - also. This effect is clearly visible in Figure 1.

#### 4. FINAL REMARKS

One of the disadvantages of the presented approach is the relatively high computational complexity, namely the fact, that the determination of the value of the described measure requires the solution of a number of, commonly non-linear, mathematical programming tasks. Although in some cases, e.g. the usability of consumption is described by the Cobb-Douglas function, the calculations, as we have shown, are considerably simpler, but still require significant computational effort. Fortunately, with the development of numerical techniques and computational power, this issue is becoming less and less important. On the other hand, the advantage of the measure in question is, that it seems to be strongly embedded in known models of spatial economics.

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